

Quantum motion of a point particle in the presence of the Aharonov-Bohm potential in curved space: viewpoint of scattering theory

Fabiano M. Andrade^a, Augusto R. Chumbes^b, Cleverson Filgueiras^c, Edilberto O. Silva^b

^a Departamento de Matemática e Estatística, Universidade Estadual de Ponta Grossa, 84030-900 Ponta Grossa, Paraná, Brazil

^b Departamento de Física, Universidade Federal do Maranhão, 65085-580 São Luís, Maranhão, Brazil

^c Departamento de Física (DFI), Universidade Federal de Lavras (UFLA), Caixa Postal 3037, 37200-000, Lavras, Minas Gerais, Brazil

Abstract

The scattering of a spinless charged particle constrained to move on a curved surface in the presence of the Aharonov-Bohm (AB) potential is studied. The particle is confined on the surface using a thin-layer procedure, which gives rise to the well-known geometric potential. We begin with the equations of motion for the surface and transverse dynamics obtained in Ref. Phys. Rev. Lett. **100** (2008) 230403 and make a connection with the description of continuous distribution of dislocations and disclinations theories. In this description, the particle is bounded to a surface with a disclination located in the $r = 0$ region. We consider the metric tensor that allows us to study the dynamics on the surface of a cone or an anti-cone. By enabling us to study the scattering problem, we chose to study the dynamics of the cone. Expressions for the modified phase shift, scattering operator and the scattering amplitude are determined by applying appropriate boundary conditions at the origin, based on the self-adjoint extension method. Finally, we find that the dependence of the scattering amplitude with energy is only due to the effects of curvature.

Keywords: Geometric potential, Self-adjoint extension, Aharonov-Bohm problem, Topological defect, Scattering operator

1. Introduction

The motion of a quantum particle constrained to move on a surface is a phenomenon that can be understood by the arising of forces that exist only as a result of the surface geometry and the quantum mechanical nature of the system. The first formalism developed in this context is based on the simulation of the classical motion of a particle on a surface in quantum mechanics by forcing the particle to move between two parallel surfaces separated by a distance d [1]. This formalism, known as thin-layer quantization, provides a result that has important physical implications in the description of the quantum mechanics of particles on surfaces. Namely, when the limit $d \rightarrow 0$ is established, one obtains an equation which differs from the usual Schrödinger equation by an additional potential which depends on the curvature of the surface. Years later, in 1981, this idea was generalized by da Costa [2], who derived the Schrödinger equation by starting from the three dimensional one and then reducing it to a two dimensional differential equation. Following this procedure, he has showed that when a quantum point particle moves confined to a surface embedded in ordinary three dimensional Euclidean space, it is subjected to a geometric potential. From his ideas, a more rigorous approach including the presence of an electric and magnetic field was proposed by Ferrari and Cuoghi [3]. They have showed that there are no couplings between the fields and the surface curvature. Moreover, by making a proper choice of the gauge, the surface and transverse dynamics are exactly separable. Such model was improved latter by considering the inclusion of the spin of the particle by Wang *et al.* [4]. Using the same thin-layer quantization scheme to constrain a quantum particle on the surface together with a transformed spinor representation, the authors have found the geometric potential and the presence of an extra factor, which can generate additional spin connection geometric potentials by the curvilinear coordinates derivatives. In a more recent work, the thin-layer quantization

Email addresses: fmandrade@uepg.br (Fabiano M. Andrade), aruedachumbes@gmail.com (Augusto R. Chumbes), cleverson.filgueiras@dfi.ufla.br (Cleverson Filgueiras), edilberto.silva@ufma.br (Edilberto O. Silva)

procedure has been refined and the procedure was further developed by taking the proper terms of degree one in q_3 (q_3 denotes the curvilinear coordinate variable perpendicular to curved surface) back into the surface quantum equation [5]. The thin-layer quantization formalism has been considered for a variety of problems with different physical contexts as, for instance, in the study of curvature effects in thin magnetic shells [6], in the quantum mechanics of a single particle constrained to move along an arbitrary smooth reference curve by a confinement that is allowed to vary along the waveguide [7], to derive the exact Hamiltonians for Rashba and cubic Dresselhaus spin-orbit couplings on a curved surface with an arbitrary shape [8], in the study of high-order-harmonic generation in dimensionally reduced systems [9], to explore the effects arising due to the coupling of the center of mass and relative motion of two charged particles confined on an inhomogeneous helix with a locally modified radius [10], to study the dynamics of shape-preserving accelerating electromagnetic wave packets in curved space [11], etc.

It is also important to mention that other alternative approaches for the confinement of a quantum particle on a surface are found in the literature. For example, in Ref. [12], a new formalism has been proposed in order to construct the Hamiltonian of a spin-1/2 particle with spin-orbit coupling confined to a surface that is embedded in a three-dimensional space spanned by a general orthogonal curvilinear coordinate. In this approach, the authors consider a gauge field that allows us to express the spin-orbit coupling as a non-Abelian $SU(2)$ gauge field. They also found that the geometric potential represents a coupling between the transverse component of the gauge field and the mean curvature of the surface that replaces the coupling between the transverse momentum and the gauge field. An extension of this approach was later accomplished in Ref. [13].

In this paper, we use the results of Ref. [3] to study the scattering process in the nonrelativistic quantum dynamics of a spinless charged particle in the presence of the AB potential [14] in curved space. This idea is a continuation of the model studied in Ref. [15] where the surface studied is described by a topological cone. In this sequel, we study the scattering problem on the cone. Using a boundary condition based on self-adjoint extension method, we obtain the modified phase shift, the S matrix and the scattering amplitude in terms of the physics of the problem. In particular, for the scattering amplitude, we verify that it depends on the energy. We compare this result with the expression for the scattering amplitude obtained in the spin-1/2 AB scattering process and verify that the origin of the dependence on energy for the scattering amplitude is due only to the effects of curvature, which arises from the geometric potential.

The plan of this work is the following. In Sec. 2, we obtain the equation that governs the motion of the particle on the cone in the presence of the AB potential. In Sec. 3, we solve the equation of motion, apply the boundary conditions allowed by the system and address the scattering problem. Expressions for the modified phase shift, S matrix, and scattering amplitude are derived. We compare our results with those obtained for the spin-1/2 AB scattering, and verify that the dependence of the scattering amplitude with energy is purely due to the curvature effects. A brief conclusion is outlined in Sec. 4.

2. Schrödinger equation for a particle on a curved surface

In this section, we write the equations that govern the motion of a spinless charged particle constrained to move on a curved surface in the presence of the Aharonov-Bohm potential. We begin by decomposing the Schrödinger equation into its normal (N) and surface (s) components [3] ($\hbar = c = 1$),

$$i\frac{\partial}{\partial t}\chi_N = \left[-\frac{\partial_3^2}{2M} + V_\lambda(q^3) \right] \chi_N, \quad (1)$$

and

$$i\frac{\partial}{\partial t}\chi_s = \frac{1}{2M} \left[-\frac{1}{\sqrt{g}}\partial_a \left(\sqrt{g}g^{ab}\partial_b \right) + \frac{iQ}{\sqrt{g}}\partial_a \left(\sqrt{g}g^{ab}A_b \right) + 2iQg^{ab}A_a\partial_b + Q^2g^{ab}A_aA_b + V_g + QV \right] \chi_s, \quad (2)$$

with $a, b = 1, 2$, where Q is the charge of the particle, A_j the covariant components of the vector potential, V_g the potential due to the geometry of the surface and V is the electric potential on the surface. Equation (1) is just an one-dimensional Schrödinger equation for a spinless particle constrained on S by the normal potential $V_\lambda(q_3)$. As we are only interested in the dynamics on the surface, Eq. (1) will be ignored in our approach. On the other hand, we can see that the Eq. (2) includes the geometrical potential V_g [16]. It is through this potential that we study the physical implications of the geometry on the dynamics of the particle.

As in Ref. [15], we make a connection with the description of continuous distribution of dislocations and disclinations in the framework of Riemann-Cartan geometry [17]. In this description, the particle is bounded to a surface with a disclination located in the $r = 0$ region. The corresponding metric tensor is defined by the line element in polar coordinates,

$$ds^2 = dr^2 + \alpha^2 r^2 d\theta^2, \quad (3)$$

with $r \geq 0$ and $0 \leq \theta < 2\pi$. For $0 < \alpha < 1$ (deficit angle), the metric (3) describes an actual cone while for $\alpha > 1$ (proficit angle), it represents an anti-cone. According to Ref. [2], the geometric potential $V_s(r)$, which is a consequence of a two-dimensional confinement on the surface, is given by

$$V_s = -\frac{1}{2M}(\mathcal{H}^2 - \mathcal{K}) = -\frac{1}{8M}(k_1 - k_2)^2, \quad (4)$$

where \mathcal{H} and \mathcal{K} are the mean and Gaussian curvature of the surface given respectively by

$$\mathcal{H} = \frac{1}{2}(k_1 + k_2) = \frac{1}{2g}(g_{11}h_{22} + g_{22}h_{11} - 2g_{12}h_{12}), \quad (5)$$

$$\mathcal{K} = k_1 k_2 = \frac{1}{g} \det(h_{ab}), \quad g = \alpha r, \quad (6)$$

where k_1 e k_2 are the principal curvatures and h_{ab} are the coefficients of the second fundamental form. In the metric (3), the geometric potential is given by an inverse squared distance potential and a δ function potential, which appear naturally in the model and depend on the type of cone [18, 19]. For the cone ($\alpha < 1$), it is given by [20]

$$\mathcal{K}_{cone} = \left(\frac{1-\alpha}{\alpha} \right) \frac{\delta(r)}{r}, \quad (7)$$

and

$$\mathcal{H}_{cone} = \frac{\sqrt{1-\alpha^2}}{2\alpha r}. \quad (8)$$

In this case, the geometric potential $V_s(r)$ reads as

$$[V_s(r)]_{cone} = \frac{1}{2M} \left[-\frac{(1-\alpha^2)}{4\alpha^2 r^2} + \left(\frac{1-\alpha}{\alpha} \right) \frac{\delta(r)}{r} \right]. \quad (9)$$

The magnetic flux tube in the background space described by the metric (3) is related to the vector potential as ($\nabla \cdot \mathbf{A} = 0, A_3 = 0$)

$$V(r) = 0, \quad -QA_i = \phi \epsilon_{ij} \frac{r_j}{\alpha r^2}, \quad (10)$$

where $\epsilon_{ij} = -\epsilon_{ji}$ with $\epsilon_{12} = +1$; $\phi = \Phi/\Phi_0$ is the flux parameter and $\Phi_0 = 2\pi/Q$. In this manner, by considering $\psi_S = e^{-iEt} \chi_S$, the Schrödinger equation (2) results in

$$-\frac{\partial^2 \chi_S}{\partial r^2} - \frac{1}{r} \frac{\partial \chi_S}{\partial r} - \frac{1}{\alpha^2 r^2} \left(\frac{\partial^2}{\partial \varphi^2} - \frac{2\phi}{i} \frac{\partial}{\partial \varphi} - \phi^2 \right) \chi_S - \frac{1-\alpha^2}{4\alpha^2 r^2} \chi_S + \left(\frac{1-\alpha}{\alpha} \right) \frac{\delta(r)}{r} \chi_S = 2ME \chi_S. \quad (11)$$

We seek solutions of the form

$$\chi_s(r, \theta) = e^{im\theta} f_m(r), \quad (12)$$

where $f_m(r)$ satisfies the eigenvalues equation ($k^2 = 2ME$)

$$hf(r) = k^2 f(r), \quad (13)$$

with

$$h = h_0 + \frac{(1-\alpha)}{\alpha} \frac{\delta(r)}{r}, \quad (14)$$

$$h_0 = -\frac{d^2}{dr^2} - \frac{1}{r} \frac{d}{dr} + \frac{\mathcal{J}_a^2}{r^2}, \quad (15)$$

where

$$\mathcal{J}_\alpha^2 = \frac{1}{4\alpha^2} \left[4(m + \phi)^2 - (1 - \alpha^2) \right], \quad (16)$$

is the effective angular momentum. Let us study (16). As pointed out in Ref. [15], for $\alpha < 1$, the quantity $(1 - \alpha^2)/\alpha^2 > 0$ and, in this case, $\mathcal{J}_\alpha^2 < 0$ for $(m + \phi)^2/\alpha^2 < (1 - \alpha^2)/4\alpha^2$, or $m + \phi = 0$. On the other hand, when $(m + \phi)^2/\alpha^2 > (1 - \alpha^2)/4\alpha^2$, we have $\mathcal{J}_\alpha^2 > 0$. This consideration ($\alpha < 1$) may also be applied in Eq. (14). In this case, $(1 - \alpha)/\alpha > 0$, in such a way that the δ function is repulsive. The case $\alpha = 1$ is not of interest because it implies a flat space. The approach considering $\mathcal{J}_\alpha^2 < 0$ implies a solution to Eq. (13) given in terms of Bessel functions of imaginary order. However, for the case $\mathcal{J}_\alpha^2 > 0$, the solution is given in terms of Bessel functions of real order. This is the reason for choosing to work with $\mathcal{J}_\alpha^2 > 0$.

3. Scattering and bound states analysis

In this section, we analyze the Aharonov-Bohm scattering in curved spacetime. It is known that the Hamiltonian h_0 is self-adjoint only for $|\mathcal{J}_\alpha| \geq 1$, whereas for $|\mathcal{J}_\alpha| < 1$ it is not self-adjoint, it has deficiency indices $(1, 1)$ and admits an one-parameter family of self-adjoint extensions [21]. Actually, h can be interpreted as a self-adjoint extension of h_0 [22]. All the self-adjoint extension of h_0 , $h_{0,\varrho}$, are accomplished by requiring the boundary condition at the origin [23],

$$\varrho_m f_{0,m}(r) = f_{1,m}(r), \quad (17)$$

with

$$\begin{aligned} f_{0,m} &= \lim_{r \rightarrow 0^+} r^{|\mathcal{J}_\alpha|} f_m(r), \\ f_{1,m} &= \lim_{r \rightarrow 0^+} \frac{1}{r^{|\mathcal{J}_\alpha|}} \left[f_m(r) - f_{0,m} \frac{1}{r^{|\mathcal{J}_\alpha|}} \right], \end{aligned}$$

where ϱ_m is the self-adjoint extension parameter. For $\varrho_m = 0$, one has the free Hamiltonian (without the δ function) with regular wave functions at the origin ($f_m(0) = 0$) while for $\varrho_m \neq 0$ the boundary condition in Eq. (17) permit a $r^{-|\mathcal{J}_\alpha|}$ singularity in the wave functions at the origin.

The general solution of the Eq. (13) for $r \neq 0$ is

$$f_m(r) = a_m J_{|\mathcal{J}_\alpha|}(kr) + b_m Y_{|\mathcal{J}_\alpha|}(kr), \quad (18)$$

where $J_\alpha(z)$ is the Bessel function of fractional order. The coefficients a_m and b_m represent the contributions of the regular and irregular solutions at the origin, respectively.

Now, we must replace the solution (18) in the boundary condition (17). Since $\lim_{r \rightarrow 0^+} r^{2-2|\mathcal{J}_\alpha|}$ is divergent if $|\mathcal{J}_\alpha| \geq 1$, then b_m must be zero. On the other hand, $\lim_{r \rightarrow 0^+} r^{2-2|\mathcal{J}_\alpha|}$ is finite for $|\mathcal{J}_\alpha| < 1$, so that there arises the contribution of the irregular solution $Y_{|\mathcal{J}_\alpha|}(kr)$. Here, the presence of an irregular solution contributing to the wave function stems from the fact the Hamiltonian h is not a self-adjoint operator when $|\mathcal{J}_\alpha| < 1$. Hence, such irregular solution must be associated with a self-adjoint extension of the operator h_0 [24, 25]. After we take into account these considerations, we get (for $|\mathcal{J}_\alpha| < 1$)

$$\frac{b_m}{a_m} = \frac{-\varrho_m k^{2|\mathcal{J}_\alpha|} \Gamma(1 - |\mathcal{J}_\alpha|) \sin(|\mathcal{J}_\alpha| \pi)}{4^{|\mathcal{J}_\alpha|} \Gamma(1 + |\mathcal{J}_\alpha|) + \varrho_m k^{2|\mathcal{J}_\alpha|} \Gamma(1 - |\mathcal{J}_\alpha|) \cos(|\mathcal{J}_\alpha| \pi)}. \quad (19)$$

Since δ function is a short range potential, it follows that the behavior of $f_m(r)$ for $r \rightarrow \infty$ is given by [26]

$$f_m(r) \sim \sqrt{\frac{2}{\pi kr}} \cos \left[kr - |m| \frac{\pi}{2} - \frac{\pi}{4} + \delta_m^{\varrho_m}(k) \right], \quad (20)$$

where $\delta_m^{\varrho_m}(k)$ is the scattering phase shift. The phase shift is a measure of the argument difference to the asymptotic behavior of the solution $J_{|m|}(kr)$ of the radial free equation that is regular at the origin. Substituting Eq. (19) and the

asymptotic behavior of $J_\nu(z)$ and $Y_\nu(z)$ [27],

$$J_\nu(kr) \approx \sqrt{\frac{2}{kr\pi}} \cos\left(kr - \frac{\nu\pi}{2} - \frac{\pi}{4}\right), \quad (21)$$

$$Y_\nu(kr) \approx \sqrt{\frac{2}{kr\pi}} \sin\left(kr - \frac{\nu\pi}{2} - \frac{\pi}{4}\right), \quad (22)$$

into the solution (18), we obtain

$$f_m(r) \sim a_m \sqrt{\frac{2}{\pi kr}} \left[\cos\left(kr - |\mathcal{J}_\alpha| \frac{\pi}{2} - \frac{\pi}{4}\right) - \frac{\varrho_m k^{2|\mathcal{J}_\alpha|} \Gamma(1 - |\mathcal{J}_\alpha|) \sin(|\mathcal{J}_\alpha| \pi) \sin\left(kr - |\mathcal{J}_\alpha| \frac{\pi}{2} - \frac{\pi}{4}\right)}{4|\mathcal{J}_\alpha| \Gamma(1 + |\mathcal{J}_\alpha|) + \varrho_m k^{2|\mathcal{J}_\alpha|} \Gamma(1 - |\mathcal{J}_\alpha|) \cos(|\mathcal{J}_\alpha| \pi)} \right]. \quad (23)$$

Comparing the expression (23) with (20), we find that the corresponding phase shift is given by

$$\delta_m^{\varrho_m}(k) = \Delta_m^{AB} + \Theta_{\varrho_m}, \quad (24)$$

where

$$\Delta_m^{AB} = \frac{\pi}{2} (|m| - |\mathcal{J}_\alpha|), \quad (25)$$

is the modified phase shift of the AB scattering, and

$$\Theta_{\varrho_m} = \arctan \left[\frac{\varrho_m k^{2|\mathcal{J}_\alpha|} \Gamma(1 - |\mathcal{J}_\alpha|) \sin(|\mathcal{J}_\alpha| \pi)}{4|\mathcal{J}_\alpha| \Gamma(1 + |\mathcal{J}_\alpha|) + \varrho_m k^{2|\mathcal{J}_\alpha|} \Gamma(1 - |\mathcal{J}_\alpha|) \cos(|\mathcal{J}_\alpha| \pi)} \right]. \quad (26)$$

It follows that the corresponding scattering operator $\mathcal{S}_m^{\varrho_m}$ (\mathcal{S} matrix) for the self-adjoint extension associated with the phase shifts (24) is given by

$$\mathcal{S}_m^{\varrho_m} = e^{2i\delta_m^{\varrho_m}(k)} = e^{2i\Delta_m^{AB}} e^{2i\Theta_{\varrho_m}}. \quad (27)$$

Using (26), Eq. (27) can be rewritten as

$$\mathcal{S}_m^{\varrho_m} = e^{2i\Delta_m^{AB}} \left[\frac{\varrho_m k^{2|\mathcal{J}_\alpha|} \Gamma(1 - |\mathcal{J}_\alpha|) e^{i\pi|\mathcal{J}_\alpha|} + 4|\mathcal{J}_\alpha| \Gamma(1 + |\mathcal{J}_\alpha|)}{\varrho_m k^{2|\mathcal{J}_\alpha|} \Gamma(1 - |\mathcal{J}_\alpha|) e^{-i\pi|\mathcal{J}_\alpha|} + 4|\mathcal{J}_\alpha| \Gamma(1 + |\mathcal{J}_\alpha|)} \right]. \quad (28)$$

Hence, for any value of the self-adjoint extension parameter ϱ_m , there is an additional scattering. If $\varrho_m = 0$, we achieve the corresponding result for the usual AB problem in curved space with Dirichlet boundary condition,

$$\mathcal{S}_m^{\varrho_m=0} = e^{i\pi(|m| - |\mathcal{J}_\alpha|)}. \quad (29)$$

We can also see that when $\alpha = 1$ in the expression for the angular momentum (16), we recover the expression for the scattering matrix obtained in Ref. [28], given by

$$\mathcal{S}_{m,\alpha=1}^{\varrho_m=0} = e^{i\pi(|m| - |m+\phi|)}. \quad (30)$$

If we assume that $\varrho_m = \infty$, we get

$$\mathcal{S}_m^{\varrho_m=\infty} = e^{2i\Delta_m^{AB} + 2i\pi|\mathcal{J}_\alpha|}. \quad (31)$$

If we set $\alpha = 1$ and $\phi = 0$ (zero magnetic flux), we have $\mathcal{S}_{m,\alpha=1}^{\varrho_m=0} = 1$ and, consequently, $f_{\varrho_m=0}^{\alpha=1}(k, \varphi) = 0$, as it should be.

We now shall discuss on the scattering amplitude $f(k, \varphi)$. This quantity can be obtained using the standard methods of scattering theory. It is given in terms of the \mathcal{S} matrix by the relation

$$f(k, \varphi) = \frac{1}{\sqrt{2\pi i k}} \sum_{m=-\infty}^{\infty} (\mathcal{S}_m^{\varrho_m} - 1) e^{im\varphi}, \quad (32)$$

with $\mathcal{S}_m^{\varrho_m}$ give in Eq. (28). Equation (32) represents the scattering amplitude for the spinless AB problem in the curved space. Note that this result differ from the usual AB scattering amplitude off a thin solenoid because it is energy dependent. For the special case when $\varrho_m = 0$, the scattering amplitude now follows from (29). As a result, we have

$$f(k, \varphi) = \frac{1}{\sqrt{2\pi i k}} \sum_{m=-\infty}^{\infty} (e^{i\pi(|m| - |\mathcal{J}_\alpha|)} - 1) e^{im\varphi}. \quad (33)$$

In the above equation we can see that the scattering amplitude differs from the usual AB scattering amplitude off a thin solenoid because it is energy dependent through the Eq. (28). Let us clarify this question. According to Ref. [29], it is known that the Dirac-Pauli wave equation

$$(\mathbf{p} - Q\mathbf{A})^2 \psi - Q(\boldsymbol{\sigma} \cdot \mathbf{B}) \psi = (E^2 - M^2) \psi = k^2 \psi, \quad (34)$$

have only spin dependence through the magnetic interaction $\boldsymbol{\sigma} \cdot \mathbf{B}$. However, as pointed out by Goldhaber [29], since a nonrelativistic particle with gyromagnetic ratio 2 obeying the Schrödinger equation,

$$(\mathbf{p} - Q\mathbf{A})^2 \chi - Q(\boldsymbol{\sigma} \cdot \mathbf{B}) \chi = 2ME\chi = k^2 \chi, \quad (35)$$

it follows that, for fixed k^2 , the large components (and separately the small components) of a solution of the Dirac equation for a nonsingular magnetic field also solve the Schrödinger equation for the same field. The scattering amplitude of a spin-1/2 particle with gyromagnetic ratio 2 interacting with a localized magnetic field is a function only of wave number k , and not explicitly dependent on mass. This is just the result of Eq. (33). For this particular case, the helicity operator associated with the Dirac particle,

$$\bar{h} = \boldsymbol{\sigma} \cdot (\mathbf{p} - Q\mathbf{A}), \quad (36)$$

and its square

$$\bar{h}^2 = (\mathbf{p} - Q\mathbf{A})^2 - Q(\boldsymbol{\sigma} \cdot \mathbf{B}), \quad (37)$$

are conserved in a static nonsingular magnetic field. For a system that does not involve a localized magnetic field configuration, the quantity $\bar{h}/\sqrt{\bar{h}^2}$ is known to be the helicity. To this physical system, the scattering amplitude for a Dirac particle on such a field configuration must be pure helicity nonflip. This result can also be interpreted through the commutation of the helicity operator with a purely Dirac Hamiltonian. On the other hand, when we consider a singular magnetic field, it fails to do so in the present case because of the $\boldsymbol{\sigma} \cdot \mathbf{B}$ term. In particular, by taking into account the field configuration (10), one obtains the localized magnetic field

$$-QB = \phi \frac{\delta(r)}{r}. \quad (38)$$

However, when we include the spin effects, this result leads to nonconservation of helicity in spin-1/2 AB scattering. Thus, we can say that the failure of helicity conservation manifested in the spin-1/2 AB problem stems from the fact that the δ function singularity make the Hamiltonian and the helicity nonself-adjoint operators [30–33]. Hence, their commutation must be analyzed carefully by considering first the correspondent self-adjoint extensions and after computing the commutation relation.

Now let us return to our problem. Contrary to the AB scattering of a particle with spin, the results above refer to a spinless particle. However, as mentioned before, the Hamiltonian (14) contains a singularity that arises from the geometric potential in the metric (3). In our case, this singularity is associated with a localized curvature, which is referred to a conical like topological defect. The singular term in Eq. (14) is equivalent to the term in Eq. (38) in the AB scattering of a particle with spin, i.e., it allows the inclusion of irregular solutions in the problem. Therefore, we can conclude that the dependence on energy in the scattering amplitude (32) is only due to the effects of localized curvature. This can be seen by taking $\alpha = 1$ in Eq. (9), which implies $V_s = 0$ and, consequently, singularities are absent.

4. Conclusions

In the present article, we have studied the scattering process of a spinless charged particle constrained to move on a cone in the presence of the AB potential. The particle is confined on the surface using the thin-layer procedure, which gives rise to the well-known geometric potential. This potential is responsible for the arising of the δ function in the equation of motion. Using a boundary condition based on self-adjoint extension method, we have obtained the modified phase shift, the S matrix and the scattering amplitude, all of them in terms of the physics of the problem. In particular, for the scattering amplitude, we verified that it depends on the energy. We compared the expression for

the scattering amplitude obtained in the spin-1/2 AB scattering process. When we are referring to scattering process of particle with spin, this dependence is a consequence of commutation of the helicity operator with the Hamiltonian. Hence, we have the helicity nonconservation. Moreover, we have verified that this conclusion can not be applied to the model addressed here because of the absence of the spin element in the system Hamiltonian with a singular magnetic field as in Eq. (38). Finally, we have argued that the origin of the dependence on energy for the scattering amplitude is due only to the effects of curvature, i.e., through the geometric potential.

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